MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

DEM5028 – ENGINEERING MATHEMATICS 2

(Diploma in Electronic Engineering)

4 JUNE 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **4** pages (**2** pages with **4** questions and **2** pages of appendix.
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

QUESTION 1 [25 MARKS]

a. Using suitable integration method, evaluate the following integrals.

i) $\int_0^1 \frac{5x^2}{2x^3+1}dx$ [6 marks]

ii) $\int \frac{x}{(x+1)(x+2)^2}dx$ [9 marks]

b. Find the area of the region bounded by the curves $y=\sqrt{x}$, $y=x$. [5 marks]

c. Find the volume of the solid generated when the region bounded by $y=x^2$, $y=0$, $x=1$ and $x=2$, is rotated about y-axis.

[5 marks]

QUESTION 2 [25 MARKS]

a. Find the solution of the differential equation $\frac{dy}{dx}=\frac{4y}{x+3}$. [4 marks]

b. For the differential equation of $\frac{dy}{dx}=3y-2e^x$, find the final solution that satisfy the condition of $y(0)=5$.

[8 marks]

c. Test either the series of $\sum_{n=1}^{\infty}\frac{2}{n\sqrt{n}}$ is converges or diverges. (Must state the test used).

[3 marks]

d. Consider the power series $\sum_{n=1}^{\infty}\frac{(x+1)^n}{n}$

i) Find the radius of convergence. [4 marks]

ii) Find the interval of convergence. [6 marks]

Continued...

QUESTION 3 [25 MARKS]

a. Consider vector $\mathbf{a} = < 1, 3 >$ and $\mathbf{b} = < 5, 2 >$, find

i) the length of the vector $\mathbf{a} - \mathbf{b}$. [4 marks]

ii) the angle between $\mathbf{a}$ and $\mathbf{b}$. [4 marks]

b. Find the area of parallelogram determined by $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

[6 marks]

c. Find the parametric and symmetric equations of the line passing through $(1, -4, 2)$ and parallel to line $x = 2 - 3t,\ y = 4 - 2t,\ z = 6 + t$.

[6 marks]

d. Find the equation of the plane through the point $(-1, 4, 5)$ and parallel to the plane $3x - 6y + 5z - 7 = 0$.

[5 marks]

QUESTION 4 [25 MARKS]

a. For $f(x, y, z) = x^2 y^3 - \frac{x}{z}$, find all its first partial derivatives. [3 marks]

b. If $z = x^2 + y^2$, where $x = u \sin v$ and $y = u - 2v$, use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$.

[10 marks]

c. A lamina occupies a region between $x = 0$, $x = 4$, $y = 0$ and $y = 3$ has a density of $\rho(x, y) = x^2 y$.

i) Find its mass. [3 marks]

ii) Find its center of mass. [9 marks]

End of page.

APPENDIX I: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

$$V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vector

The length of the vector **a** $= < a_1, a_2, a_3 >$ is $|a| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$.

If θ is the angle between the vector **a** and **b**, then $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Cross Product $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$

Equation of Line

Vector equation: $r = r_0 + tv$ Parametric equation: $x=x_0+at$, $y=y_0+bt$, $z=z_0+ct$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let

$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:** $M_x = \iint_D y\rho(x,y)dA$

The **moment** about the **y-axis:** $M_y = \iint_D x\rho(x,y)dA$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$\bar{x} = \dfrac{M_y}{m} = \dfrac{1}{m}\iint_D x\rho(x,y)dA$ $\bar{y} = \dfrac{M_x}{m} = \dfrac{1}{m}\iint_D y\rho(x,y)dA$ Where the mass: $m = \iint_D \rho(x,y)dA$

Triple Integrals: $\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$